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NONSTANDARD DISTRIBUTIONS IN TWO-STAGE LEAST SQUARES

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0. ABSTRACT

In estimating the coefficient of an endogenous variable in a single equation of a system of linear equations, Anderson and Sawa (1973) expressed the distribution of the two-stage least-squares (TSLS) estimator as a doubly noncentral F distribution. We relax their assumption of independent Gaussian errors, taking instead a scale mixture of spherical Gaussian laws in a class containing the spherical stable distributions. The resulting distribution of the TSLS estimator is a mixture of doubly noncentral F distributions mixed over the noncentrality parameters, and for suitable mixtures the normal-theory distribution is robust. Computations reported for contaminated spherical Gaussian and spherical Cauchy errors are compared with the standard case.

1. INTRODUCTION

Various procedures have been advocated for estimating the coefficents in simultaneous systems of linear equations. Arguments supporting these procedures often appeal to asymptotic properties of the estimators; in practice, however, their small-sample properties are of interest as well. A recent survey of applicable small-sample distribution theory was undertaken by Mariano (1980). One technique in wide usage is two-stage

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least-squares (TSLS) estimation under the standard assumption that errors of the model are independent, identically distributed (iid) Gaussian variables. Under these assumptions Anderson and Sawa (1973,1979) derived and later tabulated the exact distribution of the TSLS estimator for the coefficient of an endogenous variable in a single equation of a linear system. Raj (1980) studied effects of non-Gaussian errors on various estimators, including the TSLS estimator, using Monte Carlo methods. To date, however, analytical investigations of the distributions of TSLS estimators appear to have been confined to the standard case of iid Gaussian errors.

In this paper we study the exact distribution of the TSLS estimator for the coefficient of an endogenous variable in a single equation under nonstandard assumptions, replacing normality with an error distribution in the class of scale mixtures of spherical Gaussian laws (cf. Kelker (1970), for example). Our reasons for considering this class follow.

(i) Beginning with Mandelbrot (1960,1963,1969), infinite-variance distributions including the stable laws have been used to model economic variables such as stock prices; cf. also Fama (1965), McCullough (1978), Press (1975), and Samuelson (1976). The class considered here contains the spherical stable laws and other heavy-tailed distributions whose marginals figure prominently in the work cited.

(ii) The distributions of Anderson and Sawa (1973,1979) give approximations in the case of non-Gaussian errors attracted to the Gaussian law under conditions for central limit theory. The present study yields approximations in the case of errors not having second moments whose distributions are attracted to a spherical stable law.

(iii) Our approach admits a Bayesian view in which the distributions studied are posterior distributions of the TSLS estimators, the errors being conditionally iid Gaussian with a scale parameter determined by the realization of a random environment.

(iv) The ordinary least-squares estimators in a general linear model are known to retain certain optimal properties under spherical errors with and without moments (cf. Jensen (1979)).

(v) For critical linear systems statistical control charts may be devised for monitoring the stationarity of a structural relation based on Gaussian error processes and the work of Anderson and Sawa (1973). The present study lays the foundations for developing such procedures in the case of spherically invariant processes, as such processes may be represented as scale mixtures of spherical Gaussian processes (cf. Hartman and Wintner (1940)).

Our program of study follows. In Section 2 we demonstrate under scale mixtures that the distribution of the TSLS estimator is a mixture of doubly noncentral F distributions mixed over their noncentrality parameters. Evidence is given supporting the view that normal-theory distributions are robust for certain mixtures. In Section 3 selected percentiles of the TSLS estimator are presented for contaminated spherical Gaussian and spherical Cauchy errors, and these are compared with the standard distributions.

2. NONSTANDARD DISTRIBUTIONS

Let $\underset{\sim}{A}(r\times s)$ be a matrix of order $(r\times s)$; denote by $L(\underset{\sim}{Z}) = N_{r,s}(\underset{\sim}{M},\underset{\sim}{I}_r\times\underset{\sim}{\Sigma})$ that $\underset{\sim}{Z}(r\times s)$ is a random Gaussian matrix with expectation $E(\underset{\sim}{Z}) = \underset{\sim}{M}$, the rows of $\underset{\sim}{Z}$ being independent with the same dispersion matrix $\underset{\sim}{\Sigma}(s\times s)$; and let $G_{r,s}(\underset{\sim}{M},\underset{\sim}{I}_r\times\underset{\sim}{\Gamma})$ be a scale mixture of the Gaussian laws $\{N_{r,s}(\underset{\sim}{M},\tau^2(\underset{\sim}{I}_r\times\underset{\sim}{\Gamma}));\ \tau\in(0,\infty)\}$ whose probability density function (pdf) has the representation

$$g(\underset{\sim}{Y}) = (2\pi)^{-rs/2}\int_0^\infty|\tau^2\underset{\sim}{\Gamma}|^{-r/2}\exp[-\mathrm{tr}(\underset{\sim}{Y}-\underset{\sim}{M})'(\underset{\sim}{Y}-\underset{\sim}{M})\underset{\sim}{\Gamma}^{-1}/2\tau^2]dG(\tau) \qquad (2.1)$$

with $G(\tau)$ a mixing distribution on $(0,\infty)$. Further let $\chi^2(\nu,\lambda)$ be the chi-squared distribution having ν degrees of freedom and the noncentrality parameter λ, and let $F(r,s,\lambda,\eta)$ be the doubly noncentral F distribution with numerator parameters (r,λ) and denominator parameters (s,η).

Following Anderson and Sawa (1973), consider a single structural equation of the type

$$y_1 = \beta y_2 + Z_1 \gamma_1 + u \tag{2.2}$$

where y_1 and y_2 are columns of the observable matrix $Y(T\times 2)$; $Z_1(T\times K_1)$ is a matrix of rank $K_1 < T$ consisting of known exogenous variables; $\gamma_1(K_1\times 1)$ is a vector of parameters; and $u(T\times 1)$ is an error vector. The structural equation (2.2) is a member of a system of linear equations whose reduced form equations are represented by

$$Y = Z\Pi + V \tag{2.3}$$

where $Z = (Z_1, Z_2)$ is a matrix of order $(T\times K)$ and rank $K < T$ of exogenous variables; $\Pi(K\times 2)$ is a matrix of reduced form parameters partitioned as

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}$$

conformably with Z; and $V(T\times 2)$ is a matrix of random disturbances. It is assumed that (π_{21}, π_{22}) is of unit rank and that $\pi_{22} \neq 0$. Anderson and Sawa (1973) studied the k-class estimators $\hat{\beta}_k$ for β in (2.2) under the assumption that $\mathcal{L}(V) = N_{T,2}(0, I_T \times \Omega)$, with $\Omega = [\omega_{ij}]$ a (2×2) matrix.

To find the distribution of the TSLS estimator for β under scale mixtures of spherical Gaussian laws, we proceed as in Anderson and Sawa (1973, 1979) after conditioning on the mixing parameter. Their reduction to a canonical form and several subsequent transformations are valid, conditionally, for scale mixtures. Thus starting with expression (2.7) of Anderson and Sawa (1979), we obtain the following mixture representation for the cumulative distribution function (cdf) of the standardized TSLS estimator b for β, namely,

$$P\left(\frac{\delta\omega_{22}^{\frac{1}{2}}}{\sigma}(b-\beta) \leq x\right) = \int_0^\infty P\left(\frac{\delta\omega_{22}^{\frac{1}{2}}}{\sigma}(b-\beta) \leq x \mid \tau\right) dG(\tau)$$

$$= \int_0^\infty \int_{A(x)} f(\underset{\sim}{w}_1, \underset{\sim}{w}_2 \mid \tau)\, d\underset{\sim}{w}_1\, d\underset{\sim}{w}_2\, dG(\tau) \tag{2.4}$$

where

(i) α and δ are as defined in equations (2.4) and (2.8) of Anderson and Sawa (1979) with $\sigma^2 = \omega_{11} - 2\beta\omega_{12} + \beta^2\omega_{22}$ and with

$$\delta^2 = \underset{\sim}{\pi}_{22}'[Z_2'Z_2 - Z_2'Z_1(Z_1'Z_1)^{-1}Z_1'Z_2]\underset{\sim}{\pi}_{22}/\omega_{22} \tag{2.5}$$

as a noncentrality parameter;

(ii) $\underset{\sim}{w}_1$ and $\underset{\sim}{w}_2$ are random vectors of order $(K_2 \times 1)$ such that $L(\underset{\sim}{w}_1, \underset{\sim}{w}_2 \mid \tau) = N_{2K_2,1}(\underset{\sim}{\xi}, \tau^2 I_{2K_2})$ and $K_2 = K - K_1$;

(iii) $A(x) = \{(\underset{\sim}{w}_1, \underset{\sim}{w}_2) \mid \underset{\sim}{w}_1'\underset{\sim}{w}_1/\underset{\sim}{w}_2'\underset{\sim}{w}_2 \leq \theta(r)\}$ with $\theta(r) = 1 + 2r(r^2+1)^{\frac{1}{2}} + 2r^2$, where

$$r = \alpha + x(1+\alpha^2)^{\frac{1}{2}}/\delta \tag{2.6}$$

$$\alpha = \omega_{22}(\beta-\omega_{12}/\omega_{22})/|\Omega|^{\frac{1}{2}} \tag{2.7}$$

as in expressions (2.10) and (2.9), respectively, of Anderson and Sawa (1979); and

(iv) $G(\tau)$ is a mixing cdf on $(0,\infty)$.

It follows directly that

$$L(\underset{\sim}{w}_1'\underset{\sim}{w}_1/\underset{\sim}{w}_2'\underset{\sim}{w}_2 \mid \tau) = F(K_2, K_2, \lambda_1(\tau), \lambda_2(\tau)) \tag{2.8}$$

where $\lambda_1(\tau)$ and $\lambda_2(\tau)$ are the noncentrality parameters given at expressions (2.11) and (2.12) of Anderson and Sawa (1979) with δ replaced by δ/τ in their expressions (2.10) - (2.12). From our expression (2.8) it is seen that the conditioning variable τ enters the doubly noncentral F distribution only through its noncentrality parameters. The unconditional cdf of the standardized TSLS estimator thus is a mixture of doubly noncentral F distributions mixed over their noncentrality parameters, i.e.,

$$P\left(\frac{\delta\omega_{22}^{1/2}}{\sigma}(b-\beta) \leq x\right) = \int_0^\infty P(\underset{\sim}{w}_1'\underset{\sim}{w}_1/\underset{\sim}{w}_2'\underset{\sim}{w}_2 \leq \theta(r)|\tau)dG(\tau). \tag{2.9}$$

It is clear from remarks on page 704 of Anderson and Sawa (1973) that parallel results can be obtained for the ordinary least-squares (OLS) estimator b_1. Corresponding to (2.9) we have

$$P\left(\frac{\delta\omega_{22}^{1/2}}{\sigma}(b_1-\beta) \leq x\right) = \int_0^\infty P(\underset{\sim}{w}_3'\underset{\sim}{w}_3/\underset{\sim}{w}_4'\underset{\sim}{w}_4 \leq \theta(r)|\tau)dG(\tau) \tag{2.10}$$

where

$$L(\underset{\sim}{w}_3'\underset{\sim}{w}_3/\underset{\sim}{w}_4'\underset{\sim}{w}_4) = F(T-K_1, T-K_1, \lambda_1(\tau), \lambda_2(\tau)). \tag{2.11}$$

The only difference between (2.9) and (2.10) is the number of degrees of freedom of the doubly noncentral F distribution.

We conclude this section with a brief look at the possible robustness of the normal-theory distribution of the TSLS estimator against error distributions arising as scale mixtures of spherical Gaussian laws. Anderson and Sawa (1979) provided tables for the cdf of the standardized TSLS estimator in terms of the degrees of freedom (K_2) and the parameters α and δ^2 defined in (2.7) and (2.5). The values of K_2 and α are invariant under the scale change $u \to \tau u$ in the errors, whereas $\delta^2 \to \delta^2/\tau^2$. Thus the cdf of the standardized TSLS estimator under scale mixtures of Gaussian errors is a weighted average of the normal-theory cdf's corresponding to different values of δ.

A perusal of the tables of Anderson and Sawa (1979) shows that the standardized cdf often remains fairly stable over a wide range of values of

δ^2. Mixtures over these ranges accordingly should yield examples of error distributions for which the normal-theory distribution of the TSLS estimator is robust. To illustrate, consider the two-point mixing distribution $P(\tau^2=1) = p = 1 - P(\tau^2=5)$ which gives a contaminated spherical normal error distribution. If the unconditional TSLS estimator has as its parameter $\delta^2 = 5000$ with μ a fixed parameter, then its cdf is a mixture of the normal-theory cdf's corresponding to $\delta^2/\tau^2 = 1000$ when $\tau^2 = 5$ and to $\delta^2/\tau^2 = 5000$ when $\tau^2 = 1$. Selected points of the resulting cdf were evaluated for $\alpha \in \{0,1,5\}$, $K_2 \in \{3,10\}$, and $p \in \{0.50,0.75,0.90\}$ using the tables of Anderson and Sawa (1979); the results are given in Table 2.1 for $K_2 = 3$ and in Table 2.2 for $K_2 = 10$. A comparison of the normal-theory distributions as approximations to the distributions under mixtures confirms the robustness of the former under certain types of mixtures.

In the following section we undertake a broader study of the distribution of the standardized TSLS estimator under various error distributions.

3. RESULTS AND DISCUSSION

In order to compute the cdf of the standardized TSLS estimator under various choices for the mixing distribution $G(\tau)$, we follow a procedure similar to the one sketched at the end of the preceding section. First, for each fixed value of the mixing parameter τ, the cdf of the doubly noncentral F distribution is computed using the algorithm suggested by Anderson and Sawa (1979). The resulting values of these cdf's are then mixed over the relevant values of τ according to $G(\tau)$. This procedure is straightforward when $G(\tau)$ is discrete. For continuous mixtures the cdf $G(\tau)$ is discretized at the second step on partitioning $(0,\infty)$ into intervals, assigning the probabilities to mid-points of these intervals, and proceeding as for the discrete case. This procedure is expected to give good approximations to the actual distributions when the discretization is refined sufficiently.

Our numerical studies encompass three discrete mixtures of Bernoulli and binomial type giving contaminated spherical Gaussian errors, and one continuous mixture using an inverse chi distribution which gives spherical Cauchy errors. The latter is an example of a distribution having excessive tails. In particular, our Type 1 contamination uses $\{P(\tau^2=1) = 0.95, P(\tau^2=10) = 0.05\}$; Type 2 contamination uses $\{P(\tau^2=1) = 0.5, P(\tau^2=10) = 0.5\}$; and Type 3 contamination assigns probabilities to $\tau^2 \in \{1,2,\ldots,16\}$ according to the binomial distribution

$$\binom{15}{r}(0.05)^r(0.95)^{15-r} \tag{3.1}$$

with $r = \tau^2 - 1$. These discrete mixtures pose no difficulty in the computations.

The spherical Cauchy error law is a T-dimensional spherical Student's distribution having unit degree of freedom; multivariate Student's errors were considered by Zellner (1976) in connection with OLS estimation in a general linear model. Spherical Cauchy errors in TSLS estimation are obtained from $L(\tau^{-2}) = \chi^2(1,0)$ on evaluating the cdf of $\chi^2(1,0)$ over 226 intervals, assigning the probabilities of these intervals to the reciprocals of their midpoints, and then mixing the doubly noncentral F distributions over these values. As doubling the number of intervals had little effect on the values of the cdf of the TSLS estimator, the choice of 226 intervals for the discrete approximation was deemed adequate. The four types of mixtures studied in this section are summarized for convenience in Table 3.1.

Selected percentiles of the distributions of the TSLS estimators are given in Tables 3.2 - 3.7 along with the interquartile range as a measure of concentration. The range of parameters includes $\delta^2 \in \{100,300,1000,5000\}$, $\alpha \in \{0,1,5\}$, and the degrees of freedom $K_2 \in \{3,10\}$. The values tabulated by Anderson and Sawa (1979) for spherical Gaussian errors are given along with our own computations for the three contamination models and for spherical Cauchy errors. The medians and the tail probabilities reported appear to be accurate within ± 0.01, while the interquartile ranges may be overstated by an amount in the range (0.00,0.02).

Several characteristics of the TSLS estimator were noted by Anderson and Sawa (1979) in the case of iid Gaussian errors. These same tendencies are exhibited under the additional error distributions considered here. First, except when $\alpha = 0$, the TSLS estimator has negative median bias. This bias increases with α and with the degrees of freedom, K_2, but it decreases with δ^2. Second, the dispersion of the TSLS estimator, as measured by the interquartile range, decreases as α and K_2 increase, but it increases with δ^2. Together these two observations imply that as α or K_2 increase, the TSLS estimator is more tightly concentrated, but about values further removed from the true value of the parameter. The same holds true for decreasing values of δ^2.

In the preceding section we noted that the only difference between the OLS and TSLS estimators lies in the degrees of freedom of the doubly noncentral F distribution. In most practical circumstances one would expect the number of degrees of freedom for the OLS estimator to be greater than that for the TSLS estimator. From the foregoing discussion we would then expect the OLS estimator to have greater bias but smaller interquartile range than the TSLS estimator. Although the OLS estimator is not studied here numerically, our expectation agrees with results found by Raj (1980) in a Monte Carlo study using independent lognormal, uniform, and double exponential errors.

Comparing properties of the TSLS estimator under nonstandard error distributions to those for Gaussian errors in Tables 3.2 - 3.7, we note that the median bias for the nonstandard errors is never less than that for Gaussian errors. Comparisons of the interquartile ranges suggest that mixing affects location of the derived distribution of the TSLS estimator to a greater extent than scale.

For large values of the parameter δ^2 it is seen that the normal-theory distribution is reasonably robust against contaminated Gaussian errors of Types 1 and 3. Type 2 contamination and Cauchy errors yield distributions less resembling those for the standard case. From these studies it appears that rather small departures from the standard Gaussian assumptions are not crucial over a modestly wide range of the parameters of the distributions.

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TABLE 2.1. Distributions of the standardized TSLS estimator under (a) Gaussian errors with $\delta^2/\tau^2 = 1000$, (b) Gaussian errors with $\delta^2/\tau^2 = 5000$, and (c) contaminated Gaussian errors obtained as a mixture of (a) and (b) with proportions 1-p and p, respectively, for $K_2 = 3$ degrees of freedom.

x	$\alpha = 0$			$\alpha = 1$			$\alpha = 5$		
	(a)	(b)	(c) p=0.50	(a)	(b)	(c) p=0.75	(a)	(b)	(c) p=0.90
-3.00	0.0014	0.0014	0.0014	0.0009	0.0011	0.0010	0.0005	0.0009	0.0009
-2.00	0.0229	0.0228	0.0228	0.0202	0.0217	0.0213	0.0191	0.0212	0.0210
-1.00	0.1587	0.1587	0.1587	0.1664	0.1612	0.1620	0.1667	0.1623	0.1627
-0.20	0.4209	0.4210	0.4209	0.4383	0.4287	0.4311	0.4451	0.4317	0.4331
0.00	0.4997	0.4997	0.4997	0.5175	0.5077	0.5102	0.5245	0.5109	0.5123
0.20	0.5791	0.5790	0.5790	0.5960	0.5866	0.5890	0.6025	0.5809	0.5910
1.00	0.8413	0.8413	0.8413	0.8465	0.8436	0.8444	0.8485	0.8446	0.8450
2.00	0.9771	0.9772	0.9772	0.9750	0.9762	0.9759	0.9742	0.9758	0.9757
3.00	0.9986	0.9986	0.9986	0.9979	0.9983	0.9982	0.9976	0.9982	0.9981

TABLE 3.1. Description of the scale mixtures of spherical Gaussian errors studied numerically.

Type	Description
Type 1	$P(\tau^2=1) = 0.95$; $P(\tau^2=10) = 0.05$
Type 2	$P(\tau^2=1) = 0.5$; $P(\tau^2=10) = 0.5$
Type 3	$P(\tau^2=r+1) = \binom{15}{r}(0.05)^r(0.95)^{15-r}$ $r \in \{0,1,\ldots,15\}$
Cauchy	$L(\tau^{-2}) = \chi^2(1,0)$

TABLE 3.2. Selected percentiles and range of the TSLS estimator under Gaussian and various scale mixtures of Gaussian errors for K_2 = 3 degrees of freedom, α = 0, and various δ^2.

QUANTITY	Gaussian	Type 1	Type 2	Type 3	Cauchy
			ERROR DISTRIBUTIONS		
		δ^2 = 100			
Median	0.00	0.00	0.00	0.00	0.00
Interquartile Range	1.34	1.34	1.30	1.34	1.20
2.5 Percentile	-1.98	-1.98	-2.03	-1.99	-1.96
97.5 Percentile	1.98	1.98	2.03	1.99	2.00
		δ^2 = 300			
Median	0.00	0.00	0.00	0.00	0.00
Interquartile Range	1.35	1.35	1.34	1.35	1.26
2.5 Percentile	-1.97	-1.97	-1.98	-1.97	-1.96
97.5 Percentile	1.97	1.97	1.98	1.97	2.00
		δ^2 = 1000			
Median	0.00	0.00	0.00	0.00	0.00
Interquartile Range	1.35	1.35	1.35	1.35	1.30
2.5 Percentile	-1.96	-1.96	-1.97	-1.96	-1.96
97.5 Percentile	1.96	1.96	1.97	1.96	2.00
		δ^2 = 5000			
Median	0.00	0.00	0.00	0.00	0.00
Interquartile Range	1.35	1.35	1.35	1.35	1.34
2.5 Percentile	-1.96	-1.96	-1.96	-1.96	-1.96
97.5 Percentile	1.96	1.96	1.96	1.96	1.99

TABLE 3.3. Selected percentiles and range of the TSLS estimator under Gaussian and various scale mixtures of Gaussian errors for K_2 = 3 degrees of freedom, α = 1, and various δ^2.

QUANTITY	Gaussian	Type 1	Type 2	Type 3	Cauchy
			ERROR DISTRIBUTIONS		
		δ^2 = 100			
Median	-0.14	-0.16	-0.28	-0.18	-0.26
Interquartile Range	1.32	1.34	1.24	1.33	1.16
2.5 Percentile	-1.84	-1.83	-1.80	-1.81	-1.82
97.5 Percentile	2.09	2.08	2.09	2.11	2.06
		δ^2 = 300			
Median	-0.08	-0.09	-0.17	-0.10	-0.20
Interquartile Range	1.34	1.35	1.33	1.34	1.26
2.5 Percentile	-1.89	-1.88	-1.82	-1.86	-1.84
97.5 Percentile	2.04	2.04	2.08	2.05	2.06
		δ^2 = 1000			
Median	-0.05	-0.05	-0.10	-0.06	-0.14
Interquartile Range	1.35	1.36	1.35	1.35	1.31
2.5 Percentile	-1.92	-1.92	-1.88	-1.91	-1.88
97.5 Percentile	2.00	2.00	2.04	2.01	2.04
		δ^2 = 5000			
Median	-0.02	-0.02	-0.04	-0.02	-0.08
Interquartile Range	1.35	1.36	1.36	1.35	1.34
2.5 Percentile	-1.94	-1.94	-1.92	-1.94	-1.92
97.5 Percentile	1.98	1.98	2.00	1.98	2.02

TABLE 3.4. Selected percentiles and range of the TSLS estimator under Gaussian and various scale mixtures of Gaussian errors for K_2 = 3 degrees of freedom, α = 5, and various δ^2.

QUANTITY	Gaussian	Type 1	Type 2	Type 3	Cauchy
			ERROR DISTRIBUTIONS		
		δ^2 = 100			
Median	-0.19	-0.22	-0.40	-0.25	-0.38
Interquartile Range	1.30	1.32	1.17	1.31	1.12
2.5 Percentile	-1.78	-1.76	-1.68	-1.74	-1.72
97.5 Percentile	2.13	2.11	2.07	2.14	2.04
		δ^2 = 300			
Median	-0.11	-0.12	-0.23	-0.14	-0.28
Interquartile Range	1.33	1.34	1.30	1.34	1.25
2.5 Percentile	-1.86	-1.84	-1.76	-1.82	-1.78
97.5 Percentile	2.06	2.06	2.10	2.08	2.06
		δ^2 = 1000			
Median	-0.06	-0.07	-0.13	-0.08	-0.20
Interquartile Range	1.33	1.35	1.35	1.36	1.32
2.5 Percentile	-1.90	-1.90	-1.83	-1.88	-1.84
97.5 Percentile	2.02	2.02	2.06	2.03	2.06
		δ^2 = 5000			
Median	-0.03	-0.03	-0.06	-0.04	-0.10
Interquartile Range	1.35	1.35	1.35	1.36	1.35
2.5 Percentile	-1.94	-1.93	-1.90	-1.93	-1.88
97.5 Percentile	1.99	1.99	2.01	1.99	2.04

TABLE 3.5. Selected percentiles and range of the TSLS estimator under Gaussian and various scale mixtures of Gaussian errors for K_2 = 10 degrees of freedom, α = 0, and various δ^2.

QUANTITY	Gaussian	Type 1	Type 2	Type 3	Cauchy
			ERROR DISTRIBUTIONS		
		$\delta^2 = 100$			
Median	0.00	0.00	0.00	0.00	0.00
Interquartile Range	1.29	1.28	1.13	1.26	1.02
2.5 Percentile	-1.91	-1.90	-1.75	-1.88	-1.76
97.5 Percentile	1.91	1.90	1.75	1.88	1.78
		$\delta^2 = 300$			
Median	0.00	0.00	0.00	0.00	0.00
Interquartile Range	1.33	1.33	1.26	1.32	1.14
2.5 Percentile	-1.94	-1.94	-1.88	-1.93	-1.84
97.5 Percentile	1.94	1.94	1.88	1.93	1.87
		$\delta^2 = 1000$			
Median	0.00	0.00	0.00	0.00	0.00
Interquartile Range	1.34	1.34	1.32	1.34	1.24
2.5 Percentile	-1.96	-1.95	-1.93	-1.95	-1.90
97.5 Percentile	1.96	1.95	1.93	1.95	1.92
		$\delta^2 = 5000$			
Median	0.00	0.00	0.00	0.00	0.00
Interquartile Range	1.35	1.35	1.35	1.35	1.30
2.5 Percentile	-1.96	-1.96	-1.96	-1.96	-1.93
97.5 Percentile	1.96	1.96	1.96	1.96	1.96

TABLE 3.6. Selected percentiles and range of the TSLS estimator under Gaussian and various scale mixtures of Gaussian errors for K_2 = 10 degrees of freedom, α = 1, and various δ^2.

QUANTITY	Gaussian	Type 1	Type 2	Type 3	Cauchy
			ERROR DISTRIBUTIONS		
		δ^2 = 100			
Median	-0.59	-0.62	-0.89	-0.72	-0.71
Interquartile Range	1.22	1.22	1.04	1.17	1.03
2.5 Percentile	-2.19	-2.19	-2.20	-2.21	-2.14
97.5 Percentile	1.44	1.41	1.10	1.27	1.26
		δ^2 = 300			
Median	-0.36	-0.39	-0.66	-0.46	-0.58
Interquartile Range	1.30	1.31	1.23	1.29	1.17
2.5 Percentile	-2.12	-2.13	-2.20	-2.15	-2.13
97.5 Percentile	1.69	1.67	1.42	1.60	1.52
		δ^2 = 1000			
Median	-0.20	-0.22	-0.40	-0.26	-0.42
Interquartile Range	1.33	1.34	1.32	1.34	1.30
2.5 Percentile	-2.06	-2.07	-2.13	-2.08	-2.10
97.5 Percentile	1.83	1.82	1.67	1.78	1.71
		δ^2 = 5000			
Median	-0.09	-0.10	-0.19	-0.12	-0.26
Interquartile Range	1.35	1.35	1.35	1.35	1.36
2.5 Percentile	-2.01	-2.02	-2.05	-2.02	-2.06
97.5 Percentile	1.91	1.90	1.84	1.88	1.85

TABLE 3.7. Selected percentiles and range of the TSLS estimator under Gaussian and various scale mixtures of Gaussian errors for $K_2 = 10$ degrees of freedom, $\alpha = 5$, and various δ^2.

QUANTITY	Gaussian	Type 1	Type 2	Type 3	Cauchy
			ERROR DISTRIBUTIONS		
		$\delta^2 = 100$			
Median	-0.82	-0.86	-1.28	-1.00	-1.02
Interquartile Range	1.15	1.16	0.92	1.08	1.05
2.5 Percentile	-2.24	-2.24	-2.25	-2.26	-2.20
97.5 Percentile	1.19	1.16	0.82	0.97	1.04
		$\delta^2 = 300$			
Median	-0.50	-0.54	-0.94	-0.62	-0.83
Interquartile Range	1.28	1.30	1.20	1.26	1.22
2.5 Percentile	-2.17	-2.18	-2.24	-2.20	-2.18
97.5 Percentile	1.58	1.55	1.26	1.44	1.40
		$\delta^2 = 1000$			
Median	-0.28	-0.32	-0.56	-0.36	-0.58
Interquartile Range	1.33	1.34	1.32	1.32	1.36
2.5 Percentile	-2.09	-2.10	-2.18	-2.12	-2.16
97.5 Percentile	1.77	1.76	1.56	1.71	1.63
		$\delta^2 = 5000$			
Median	-0.13	-0.14	-0.26	-0.16	-0.34
Interquartile Range	1.34	1.35	1.35	1.34	1.42
2.5 Percentile	-2.03	-2.03	-2.09	-2.04	-2.10
97.5 Percentile	1.88	1.88	1.80	1.86	1.81

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The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Simultaneous equations; two-stage least squares; spherical error distributions; mixtures of distributions; robustness.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In estimating the coefficient of an endogenous variable in a single equation of a system of linear equations, Anderson and Sawa (1973) expressed the distribution of the two-stage least-squares (TSLS) estimator as a doubly noncentral F distribution. We relax their assumption of in-

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dependent Gaussian errors, taking instead a scale mixture of spherical Gaussian laws in a class containing the spherical stable distributions. The resulting distribution of the TSLS estimator is a mixture of doubly noncentral F distributions mixed over the noncentrality parameters, and for suitable mixtures the normal-theory distribution is robust. Computations reported for contaminated spherical Gaussian and spherical Cauchy errors are compared with the standard case.

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